Representations of lens spaces

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Abstract

Lens spaces are the simplest family of closed 3-manifolds; they may be defined in different ways. In this talk the focus will be on the genus one Heegaard splitting definition: a lens space is the glueing of the two solid tori through an homeomorphism of their boundaries that can be represented by an element of $SL(2,\mathbb{Z})$.

Lens spaces may be defined also by a quotient of the 3-sphere \mathbf{S}^3 under the action of \mathbb{Z}_p , by a glueing of the boundary of the ball B^3 and by integral or rational Dehn surgery on knots/links. The equivalence of all these definitions can be easily described through some pictures.

At last, the classification results of lens spaces will be outlined, both up to diffeomorphism and up to homotopy type.

If time will permit, it will be shown that every closed 3-manifold may be described by an Heegaard splitting, as well as Dehn surgery on knots/links.