Gromov-Witten Invariants

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Abstract

What are they?

In algebraic geometry they represent the intersection theory on moduli spaces of pseudo-holomorphic curves in almost Kähler manifolds.

In mathematical physics they are symplectic invariants that can be read as coefficients in the multiplication table of the quantum cohomology ring of homogeneous varieties.

In A-model string theory they represent well defined path integrals of the theory, and they play a fundamental role in the Mirror Symmetry for Calaby Yau manifolds.

Yes, but how can I compute them?

They are usually hard to compute, we will go through some examples:

1. $GW(\{pt\})$ and the Witten conjecture
2. $GW(\mathbb{P}^1)$ and the recently found quantum curve
3. $GW(\mathbb{P}^2)$ and the amazing Kontsevich-Manin Formula
4. Other intriguing examples, like the comparison between the quantum cohomologies of $\mathbb{P}^3$ and a smooth quadric 3-fold that have very similar classical cohomology rings.

References

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